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CHAPTER - 1
RELATIONS AND FUNCTIONS

★ RELATIONS' TYPES

~~A=~~ ~~{~~

- **REFLEXIVE RELATION**

$(a,a) \in R \ , \ \forall a \in A$

(i) $A = \{2,3,4\}$

$R = \{(x,y) : x \text{ divides } y \ \& \ x,y \in A\}$

$R = \{(2,2), (3,3), (4,4), (2,4)\}$

$\because (a,a) \in R \quad \forall a \in A$

So, R is reflexive.

(ii) A = Set of lines in a plane

R = is $\perp$ to

No line is perpendicular to itself

$(l_1, l_1) \notin R$

Every identity relation is reflexive but every reflexive relation is not identity relation.

- **SYMMETRIC RELATION**

$(a,b) \in R \Rightarrow (b,a) \in R \ , \ a,b \in A$

R is symmetric

$(a,b) \in R$ but $(b,a) \notin R \ , \ a,b \in A$

R is not symmetric

(i) A = Set of human beings

$R = \{(x,y) : x \text{ and } y \text{ live in same locality}\}$

Let $(x,y) \in R \Rightarrow x$ and y live in same locality

$\Rightarrow$ y and x live in same locality

$\Rightarrow (y, x) \in R \quad , \quad x, y \in A$

- TRANSITIVE RELATION

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R, \qquad a, b, c \in A$

(i) A = set of all lines in a plane

R = is parallel to

$(l_1, l_2) \in R, (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2$ and $l_2 \parallel l_3$

$\Rightarrow l_1 \parallel l_3$

$\Rightarrow (l_1, l_3) \in R \longrightarrow$ ~~not~~ transitive

* A relation which is reflexive, symmetric and transitive is called equivalence relation.

(ii) A = set of all lines in a plane

R = is $\perp$ to

$(l_1, l_2) \in R, (l_2, l_3) \in R \Rightarrow l_1 \perp l_2$ and $l_2 \perp l_3$

But l_1 is not perpendicular to l_3

R is not transitive.

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★ EXERCISE - 1.1

Q1.) Determine whether each of the following relation are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as
$$R = \{(x, y) : 3x - y = 0\}$$

(i) $A = \{1, 2, \ldots, 14\}$
$R = \{(x, y) : 3x - y = 0\}$
$$3x - y = 0$$
$$\Rightarrow y = 3x$$
$$y = 3 \times 1 = 3$$
$$y = 3 \times 2 = 6$$
$$y = 3 \times 3 = 9$$
$$y = 3 \times 4 = 12 \; \vdots \ldots$$

$R = \{(1,3), (2,6), (3,9), (4,12)\}$

Reflexive
- $(1,1) \notin R, \; 1 \in A$

R is not reflexive
$\because (a,a) \notin R, \; \forall a \in A$

Symmetric
- $(a,b) \in R$ but $(b,a) \notin R$

R is not symmetric
$\because (1,3) \in R$ but $(3,1) \notin R$

Transitive
$(a,b) \in R, (b,c) \in R$ but $(a,c) \notin R$
$\because (1,3) \in R, (3,9) \in R$ but $(1,9) \notin R$
R is not transitive

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as
$$R = \{(x, y) : y \text{ is divisible by } x\}$$

(iii) Reflexive

Every no. is divisible by itself
$(a, a) \in R$, $\forall a \in A$
R is reflexive.

Symmetric

$(a, b) \in R$ but $(b, a) \notin R$
$(2, 4) \in R$ but $(4, 2) \notin R \longrightarrow$ not symmetric

Transitive

Let $(a, b) \in R$ and $(b, c) \in R \Rightarrow b = \lambda a$ and $c = \mu b$ [λ and μ are +ve int.]
$\Rightarrow c = \mu \times \lambda a$
$\Rightarrow c = (\lambda\mu)a$
$\Rightarrow (a, c) \in R \longrightarrow$ transitive

(ii) Relation R in the set N of natural numbers defined as
$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

(ii) $x = 1, 2, 3$
$y = x + 5$
$y = 1 + 5 \Rightarrow 6$
$y = 2 + 5 \Rightarrow 7$
$y = 3 + 5 \Rightarrow 8$

$R = \{(1, 6), (2, 7), (3, 8)\}$

Reflexive

$(1, 1) \notin R$, $1 \in A$
$\because (a, a) \notin R$, $\forall a \in A$
R is not reflexive

Symmetric

$(1,6) \in R$ but $(6,1) \notin R$

$\because (a,b) \in R$ but $(b,a) \notin R$

R is not symmetric

Transitive

$\because (a,b) \in R$ but $(b,c) \notin R \Rightarrow (a,c) \notin R$

R is not transitive

(iv) Relation R in the set Z of all integers defined as

$$R = \{(x,y) : x-y \text{ is an integer}\}$$

(iv) Reflexive

$\because (x,x) \in R \Rightarrow x-x$ is an integer, $\forall x \in Z$

R is reflexive

Symmetric

$(x,y) \in R \Rightarrow x-y$ is an integer

$\Rightarrow y-x$ is also an integer

$\Rightarrow (y,x) \in R$

R is symmetric

Transitive

Let $(x,y) \in R \Rightarrow x-y$ — ① is an integer

and $(y,z) \in R \Rightarrow y-z$ — ② is an integer

① + ②

$(x-y)+(y-z)$

$\Rightarrow x-z$ is also an integer since addition of two integers is an integer

$(x,z) \in R$

R is transitive

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x,y) : x$ and y work at the same place$\}$

(a) ① $(x,x) \in R \Rightarrow x$ belongs to a town and works at same place as himself

$\therefore R$ is reflexive

② $(x,y) \in R \Rightarrow x$ and y work at same place

$\Rightarrow y$ and x work at same place

$\Rightarrow (y,x) \in R$

$\therefore R$ is symmetric

③ Let $(x,y) \in R$ and $(y,z) \in R$

x and y work at same place, and y and z also work at same place

Then, x and z must also work at same place

$(x,z) \in R$

$\therefore R$ is transitive

(b) $R = \{(x,y) : x$ and y live in the same locality$\}$

(b) ① $(x,x) \in R \Rightarrow x$ lives in same locality as himself

$\therefore R$ is reflexive

② $(x,y) \in R \Rightarrow x$ and y live in the same locality

$\Rightarrow y$ and x live in the same locality

$\Rightarrow (y,x) \in R$

$\therefore R$ is symmetric

③ Let $(x,y) \in R$ and $(y,z) \in R$

x and y live in same locality, and y and z live in same locality

Then, x and z must also live in same locality

$(x,z) \in R$

$\therefore R$ is transitive

(c) $R = \{(x,y) : x$ is exactly 7 cm taller than $y\}$

(c) ① $(x, x) \notin R$

No one is taller than himself

$\therefore$ R is not reflexive

② $(x, y) \in R$ but $(y, x) \notin R$

Two ~~no~~ ~~one~~ person can ~~be~~ never be taller than each other

$\therefore$ R is not symmetric

③ Let $(x, y) \in R$ and $(y, z) \in R$

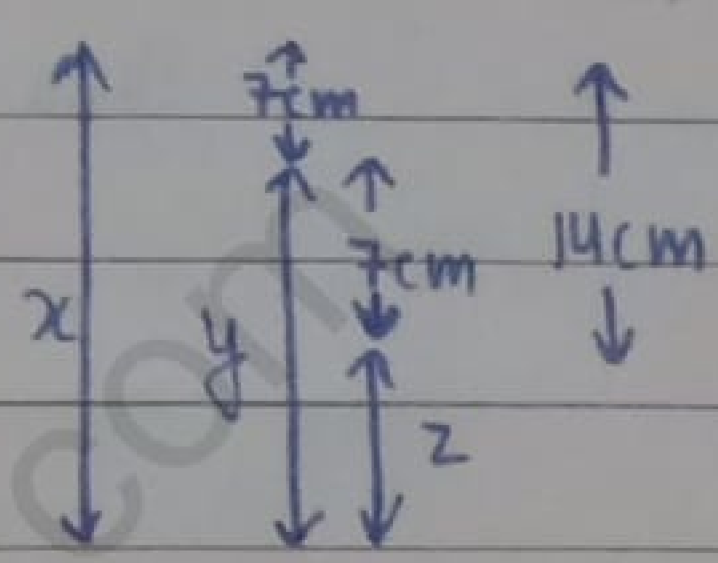
x is exactly 7cm taller than y

y is exactly 7 cm taller than z

Then, x is 14 cm taller than z

$(x, z) \notin R$

$\therefore$ R is not transitive



(d) $R = \{(x, y) : x$ is wife of $y\}$

(d) ① $(x, x) \notin R$

No one can be wife of herself

$\therefore$ R is not reflexive

② $(x, y) \in R$ but $(y, x) \notin R$

If x is wife of y then y will be the husband of x

$\therefore$ R is not symmetric

③ $(x, y) \in R$ but $(y, z) \notin R \Rightarrow (x, z) \notin R$

If x is wife of y then y will be the husband of x and y cannot be the wife of z

$\therefore$ R is not transitive

(e) $R = \{(x, y) : x$ is father of $y\}$

(e) ① $(x, x) \notin R$

No one can be father of himself

$\therefore$ R is not reflexive.

② $(x,y) \in R$ but $(y,x) \notin R$
If x is the father of y then y will be the son/daughter of x
$\therefore$ R is not symmetric

③ $(x,y) \in R$ but $(y,z) \in R \Rightarrow (x,z) \notin R$
If x is father of y and y is father of x
then x will be grand father of z
$\therefore$ R is not transitive.

Q4.) Show that the relation R in the set R of real numbers, defined as $R = \{(a,b) : a \le b\}$ is reflexive and transitive but not symmetric.

Sol. ① $(a,a) \in R$, $\forall a \in R$
$a \le a$
$\therefore$ R is reflexive

② $a \le b$ but $b \le a$
$(a,b) \in R$ but $(b,a) \notin R$
$1 \le 2$ but $2 \nleq 1$
$\therefore$ R is not symmetric

③ Let $(a,b) \in R$ and $(b,c) \in R$
$a \le b$ and $b \le c$
Then $a \le c \Rightarrow (a,c) \in R$
$\therefore$ R is transitive

Q2.) Show that the relation R in the set R of real numbers, defined as $R = \{(a,b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.

Sol. ① $\frac{1}{2} \nleq \left(\frac{1}{2}\right)^2$

$\left(\frac{1}{2}, \frac{1}{2}\right) \notin R \Rightarrow (a,a) \notin R, \ \forall a \in R$

$\therefore$ R is not reflexive

② $(1,2) \in R$

$1 \leq 2^2$

but $2 \nleq 1^2$

$(2,1) \notin R$

$(a,b) \in R$ but $(b,a) \notin R$

$\therefore$ R is not symmetric

③ Let ~~(2,3)~~ $(2,-3) \in R$ and $(-3,1) \in R$

$2 \leq (-3)^2$

$(-3) \leq 1^2$

but $2 \nleq 1^2$

$(2,1) \notin R$

$(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \notin R$

$\therefore$ R is not transitive.

Q5) Check whether the relation R in R defined ~~by~~ by $R = \{(a,b) : a \leq b^3\}$ is reflexive, symmetric or ~~the~~ transitive.

Sol. ①

$\frac{1}{2} \nleq \left(\frac{1}{2}\right)^3$

$\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$

$(a,a) \notin R, \ \forall a \in R$

$\therefore$ R is not reflexive

② $(-2,3) \in R$

$-2 \le 3^3$

but $3 \not\le (-2)^3$

$(3,-2) \notin R$

$(a,b) \in R$ but $(b,a) \notin R$

$\therefore$ R is not symmetric

③ Let $(16,4) \in R$ and $(4,2) \in R$

$16 \le 4^3$ $\quad$ $4 \le 2^3$

$\Rightarrow 16 \le 64$ $\quad$ $\Rightarrow 4 \le 8$

But, $16 \not\le 2^3$

$\Rightarrow 16 \not\le 8$

$(16,2) \notin R$

$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \notin R$

$\therefore$ R is not transitive

Q3.) Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is reflexive, symmetric or transitive.

Sol. $A = \{1,2,3,4,5,6\}$

$R = \{(a,b) : b = a+1\}$

$R = \{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)\}$

① $(1,1) \notin R$

$(a,a) \notin R, \forall a \in A$

$\therefore$ R is not reflexive

② $(1,2) \in R$

but $(2,1) \notin R$

$(a,b) \in R$ but $(b,a) \notin R$

$\therefore$ R is not symmetric

③ ~~Let~~ $(1,2) \in R$ and $(2,3) \in R$

but $(1,3) \notin R$

$(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \notin R$

$\therefore$ R is not transitive

EXTRA QUESTION

Q Prove that relation R on set Z of all integers defined by $(x,y) \in R \Leftrightarrow (x-y)$ is divisible by n is an equivalence relation on Z.

Sol. ① $x-x=0$ is divisible by n

$(x,x) \in R, \forall x \in Z$

$\therefore$ R is reflexive.

② $(x,y) \in R \Rightarrow x-y$ is divisible by n

$\Rightarrow x-y = nq, \quad q \in Z$

$\Rightarrow -(y-x) = nq$

$\Rightarrow y-x = n(-q)$

$\Rightarrow (y,x) \in R$

$\therefore$ R is symmetric

③ $(x,y) \in R$ and $(y,z) \in R$

$x-y = np$ — Ⓐ $\quad y-z = nq$ — Ⓑ $\quad , p,q \in Z$

Ⓐ + Ⓑ $\Rightarrow x-y+y-z = np+nq$

$\Rightarrow x-z = n(p+q)$

$\Rightarrow (x,z) \in R$

[Sum of two integers is also an integer $\therefore (p+q) \in Z$]

$\therefore$ R is transitive

R is reflexive, symmetric and transitive

$\therefore$ It is an equivalence relation

Hence, proved

~~EXTRA QUESTION~~

Q9.) (i) Show that the relation R on a set $A = \{x : x \in Z, 0 \leq x \leq 12\}$ given by $R = \{(a,b) : |a-b|$ is a multiple of $4\}$ is an equivalence relation

(i) $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

① $|a-a| = 0$ is a multiple of 4
$(a,a) \in R$, $\forall a \in A$
$\therefore R$ is reflexive.

② Let $(a,b) \in R \Rightarrow |a-b|$ is a multiple of 4
~~$\Rightarrow |-(b-a)|$ is a multiple of 4~~
$\Rightarrow |b-a|$ is a multiple of 4 $[\because |a-b| = |b-a|]$
$\Rightarrow (b,a) \in R$
$\therefore R$ is symmetric

③ Let $(a,b) \in R$ and $(b,c) \in R \Rightarrow a-b = \pm 4\lambda$, $b-c = \pm 4\mu$, $\lambda, \mu \in Z$
$\Rightarrow a-b+b-c = \pm 4(\lambda+\mu)$
$\Rightarrow a-c = \pm 4(\lambda+\mu)$
$\Rightarrow |a-c|$ is a multiple of 4
$\Rightarrow (a,c) \in R$
$\therefore R$ is transitive

R is reflexive, symmetric and transitive
$\therefore R$ is an equivalence relation.
Hence, proved

~~EXTRA QUESTION~~

Find the set of all elements related to 1.
$|1-1| = 0$ is a multiple of 4
$[1] = \{1, 5, 9\}$

☆ A is any set, R is a relation on A

$[x] = \{y : (x, y) \in R\}$

EXTRA QUESTION

Q Prove that the relation R on the set $N \times N$ defined by $(a, b)\, R\, (c, d) \Leftrightarrow a+d = b+c \;\forall\; (a, b), (c, d) \in N \times N$ is an equivalence relation.

Sol. ① Let $(a, b) \in N \times N$

$(a, b)\, R\, (a, b) \in R \Rightarrow$ ~~$a+b = a+b$~~ $a+b = b+a$ [~~which is true since~~ ∵ Sum of natural numbers is commutative]

~~$\Rightarrow (a, b)\, R\, (a, b)$~~

$\therefore$ R is reflexive.

② $(a, b)\, R\, (c, d) \Rightarrow a+d = b+c$ [∵ Sum of natural numbers is commutative]

$\Rightarrow b+c = a+d$

~~$(c, d)\, R\, (a, b)$~~ $\Rightarrow c+b = d+a$

$\Rightarrow (c, d)\, R\, (a, b)$

$\therefore$ R is symmetric

③ $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

$\Rightarrow a+d = b+c$ and $c+f = d+e$

$\Rightarrow a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$

$\Rightarrow a+f = b+e$

$\Rightarrow (a, b)\, R\, (e, f)$

$\therefore$ R is transitive.

R is reflexive, symmetric and transitive

$\therefore$ R is an equivalence relation

Hence, proved

● EXTRA QUESTION

Q Prove that the relation $a \equiv b(m)$ of all the integers is an equivalence relation.

Sol. ① $a-a=0$ is divisible by m

$a \equiv a(m)$

$\therefore R$ is reflexive.

② $(a,b) \in R \Rightarrow (a-b) = \lambda m, \quad \lambda \in Z$

$\Rightarrow (b-a) = (-\lambda)m, \quad -\lambda \in Z$

$\Rightarrow (b,a) \in R$

$\therefore R$ is symmetric

③ $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a-b) = \lambda_1 m$ and $(b-c) = \lambda_2 m$, $\lambda_1, \lambda_2 \in Z$

$\Rightarrow a-b+b-c = (\lambda_1+\lambda_2)m$

$\Rightarrow a-c = (\lambda_1+\lambda_2)m$

$\Rightarrow (a,c) \in R$ $\quad \therefore R$ is an equivalence relation

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Q12.) Show that the relation R defined in the set of all triangles as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, ~~with~~ T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Sol. $R = \{(T_1, T_2): T_1 \sim T_2\}$

① $\because T_1 \sim T_1$, every triangle is similar to itself

$(T_1, T_1) \in R, \forall T_1 \in A$ $\quad \therefore R$ is reflexive

② $(T_1, T_2) \in R \Rightarrow T_1 \sim T_2$

$\Rightarrow T_2 \sim T_1$

$\Rightarrow (T_2, T_1) \in R$ $\quad \therefore R$ is symmetric

③ $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$

$\Rightarrow T_1 \sim T_2$ and $T_2 \sim T_3$

$\Rightarrow T_1 \sim T_3$

$\Rightarrow (T_1, T_3) \in R$ $\therefore$ R is transitive

R is reflexive, symmetric and transitive

$\therefore$ R is an equivalence relation

Now,	T_1	3, 4, 5
	T_2	5, 12, 13
	T_3	6, 8, 10

$$\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$$

$T_1 \sim T_3$

EXTRA QUESTION

Q If R and S are equivalence relation on set A, then show that $R \cap S$ is an equivalence relation.

Sol. $R \subset A \times A$, $S \subset A \times A$

$R \cap S \subset A \times A$

① $(a,a) \in R$, $(a,a) \in S$

$(a,a) \in R \cap S$

$\therefore$ R is reflexive

② $(a,b) \in R \cap S \Rightarrow (a,b) \in R$ and $(a,b) \in S$

$\Rightarrow (b,a) \in R$ and $(b,a) \in S$

$\Rightarrow (b,a) \in R \cap S$

$\therefore$ R is symmetric

③ $(a,b), (b,c) \in R \cap S \Rightarrow (a,b) \in R$, $(b,c) \in R$, $(a,b) \in S$ and $(b,c) \in S$

$\Rightarrow (a,c) \in R$ and $(a,c) \in S$

$\Rightarrow (a,c) \in R \cap S$

$\therefore$ R is transitive

$R \cap S$ is reflexive, $R \cap S$ symmetric and transitive

$\therefore$ $R \cap S$ is an equivalence relation

EXTRA QUESTION

Q Prove that the relation R on the set $N \times N$ defined by $(a,b) R (c,d) \Leftrightarrow ad = bc \;\forall\; (a,b), (c,d) \in N \times N$ is an equivalence relation.

Sol. ① Let $(a,b) \in N \times N$ ~~ab = ab~~

$(a,b) R (a,b) \in R \Rightarrow ab = ba$ which is true since [Product of natural numbers is commutative]

~~$\Rightarrow (a,b) R (a,b)$~~

$\therefore$ R is reflexive

② $(a,b) R (c,d) \Rightarrow ad = bc$

$\Rightarrow da = cb$ [Product of natural numbers is commutative]

$\Rightarrow cb = da$

$\Rightarrow (c,d) R (a,b)$

$\therefore$ R is symmetric

③ $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow ad = bc$ and $cf = de$

$\because d = \frac{bc}{a}$

$\Rightarrow \boxed{\cancel{ad = bc} \text{ and}}\; \cancel{c}f = \frac{b\cancel{c}e}{a}$

$\Rightarrow \boxed{\cancel{ad = bc} \text{ and}}\; af = be$

$\Rightarrow (a,b) R (e,f)$

$\therefore$ R is transitive

R is reflexive, symmetric and transitive

$\therefore$ R is an equivalence relation.

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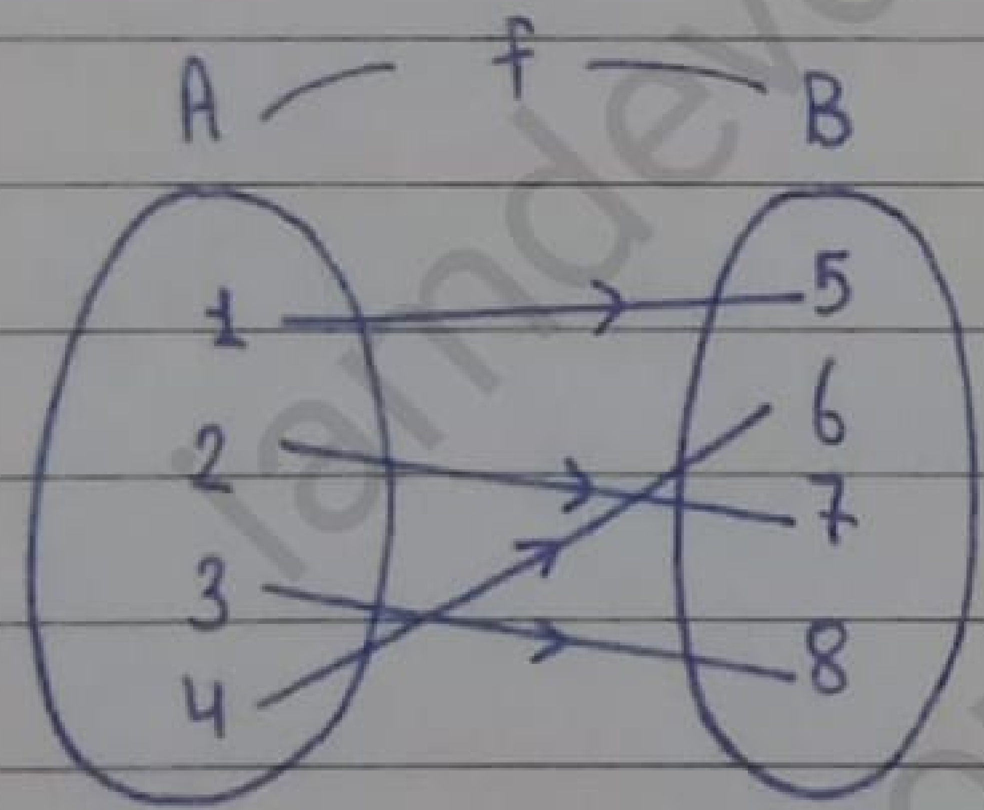
★ FUNCTIONS → A subset f of $A \times B$ is said to be a function from A to B if every element of set A has unique image in

- One - One (Injective)
- Many - One
- Onto (Surjective)
- Into

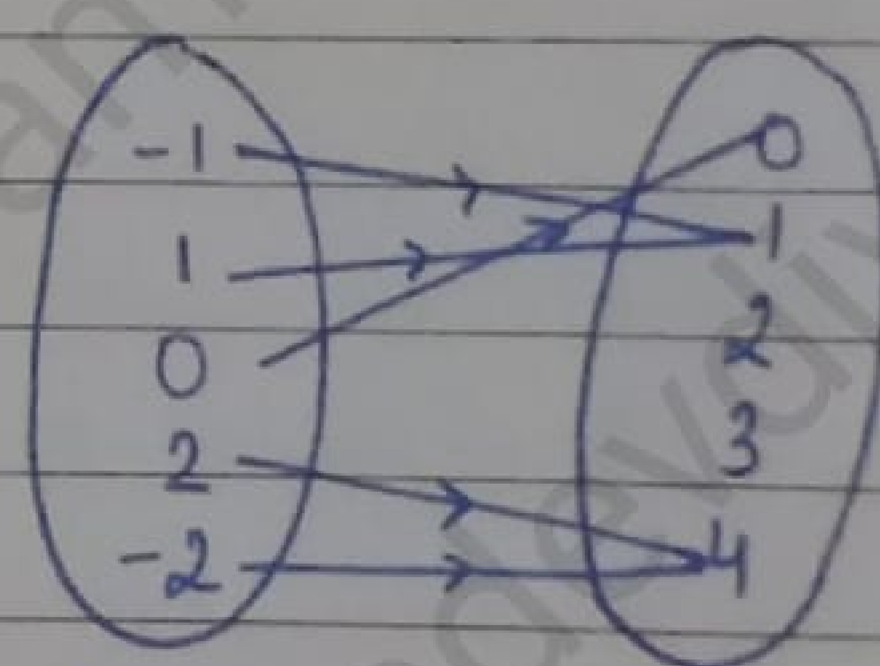
One-One + Onto → Bijective

Types of Functions

(i) $f: A \rightarrow B$ is a one-one function if every element of set A has distinct images in B under f. Otherwise, it is called many-one function.



One-one (injective)



Many-one

- EXAMPLE QUESTIONS

Q Is the given function one-one or many-one?

$f: N \rightarrow N$

$f(x) = x^2$

Sol. Let $x_1, x_2 \in N_D$ (Domain)

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore$ F is one-one

Q Is the given function one-one or many-one?

~~Sol.~~ $f : R \rightarrow R$

$f(x) = x^2$

Sol. Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

$x_1 = x_2 \; , \; x_1 = -x_2$

$\therefore$ f is many-one

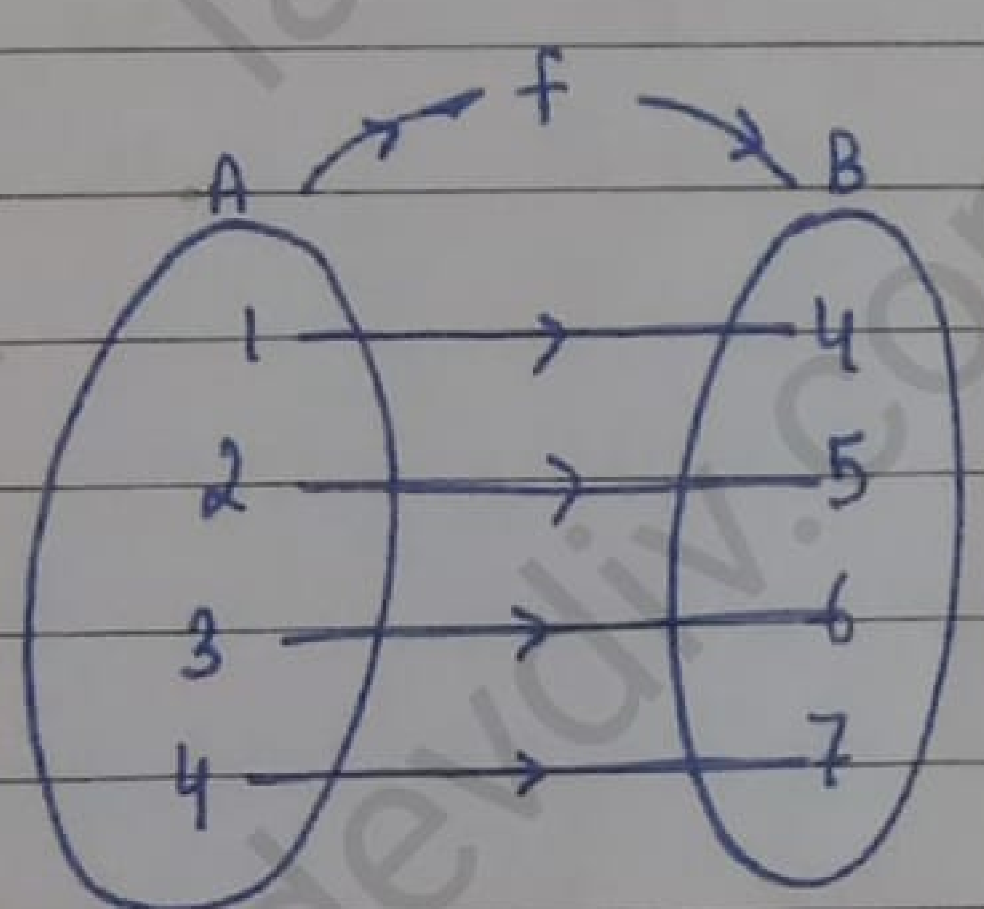
(ii) Onto function $\longrightarrow$ $f : A \rightarrow B$

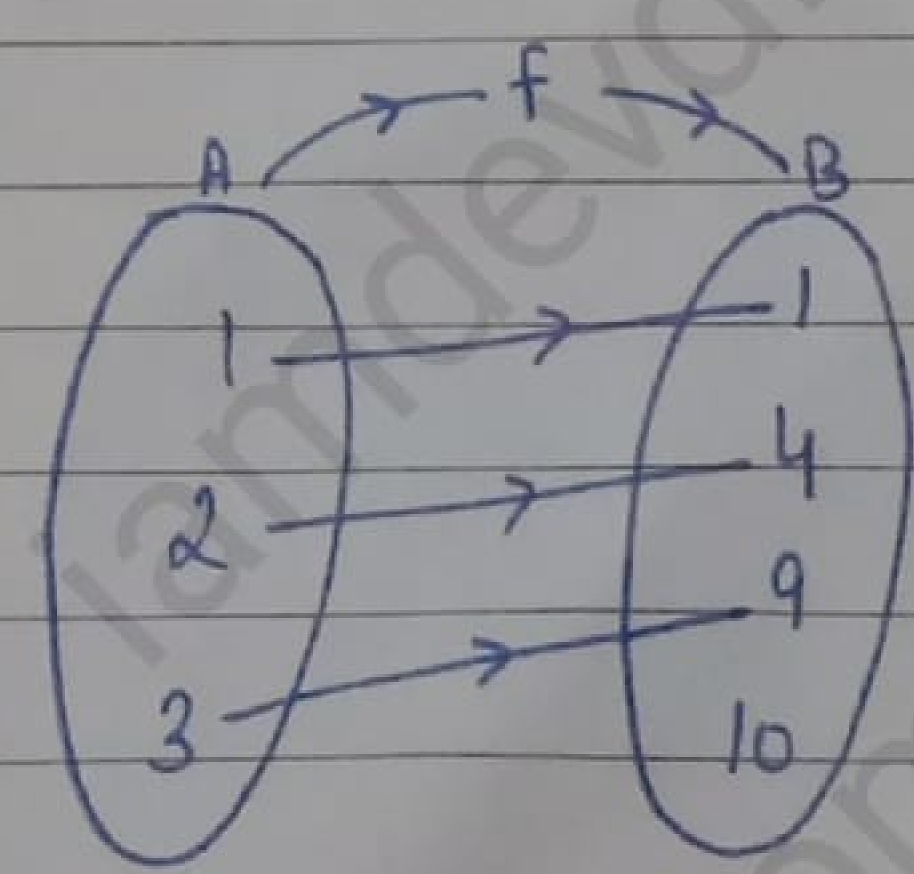
Range of f = B (codomain)

(iii) Into function $\longrightarrow$ $f : A \longrightarrow B$

Range of $f \subset B$ (codomain)

(iv) Bijective $\rightarrow$ A one-one onto function is bijective.





$y = f(x) = x + 3$

Range of f = B

$\therefore$ ~~onto~~ f is onto function (surjective)

$f(x) = x^2$

Range of $f \subset B$

$\therefore$ f is into function

- EXAMPLE QUESTIONS

Q Let $f(x) = x^2$, $f : N \longrightarrow N$, is f into or onto function?

Sol. Let y be any arbitrary element of N (codomain)

$f(x) = y \in N$

$\Rightarrow x^2 = y$

$\Rightarrow x = \sqrt{y}$

Let $y = 2$, $x = \sqrt{2} \notin N$ (domain)

Thus, for every $y \in N_c$, $x = \sqrt{y} \notin N$ (domain)

$\therefore$ f is into function

Q Let $f: R \rightarrow R$, $f(x) = 4x+3$. Prove that f is onto function.

Sol. Let $y \in R$ (c)

such that $f(x) = y = 4x+3$

$$\Rightarrow x = \frac{y-3}{4}$$

For every $y \in R$ $\exists$ an element $x = \frac{y-3}{4} \in R$ (domain)

↳ there exists

such that $f\left(\frac{y-3}{4}\right) = \cancel{4} \times \frac{y-3}{\cancel{4}} + 3$

$= y$

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★ EXERCISE - 1.2

Q2) Check the injectivity and surjectivity of the following functions:

(ii) $f: Z \longrightarrow Z$ given by $f(x) = x^2$

(ii) One-one

$x_1, x_2 \in Z$

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

$x_1 = x_2$

$x_1 = -x_2$

f is not ~~on~~ injective.

Onto

Let $y \in Z_C$

such that $f(x) = y$

$\Rightarrow x^2 = y$

$\Rightarrow x = \sqrt{y} \in Z_D$

$2 \in Z_C$ but $\sqrt{2} \in Z_D$

f is not surjective.

Q3.) Prove that the Greatest Integer Function $f: R \rightarrow R$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Sol. One-one

$1.2, 1.3 \in R_D$

$[1.2] = 1$

$[1.3] = 1$

Two elements have same image
so $\therefore$ f is not one-one.

Onto

$\because$ Range of greatest integer function is Z.
$Z \subset R_c$
Range $\subset$ Codomain
So f is not onto.

Q9.) ~~State~~ Let $f: N \longrightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer.

Sol. $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

$f(1) = \frac{1+1}{2} = \frac{2}{2} = 1$

$f(2) = \frac{2}{2} = 1$

Two elements have same image so f is not one-one
$\therefore$ f is not bijective

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★ EXERCISE - 1.1 (Continue)

Q6.) Show that the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive.

Sol. ① $(1,1) \notin R \ \forall, 1 \in R$

$(a,a) \notin R \ \forall, a \in R$

$\therefore$ R is not reflexive

② $(1,2) \in R$ but $(2,3) \notin R$

$(a,b) \in R$ but $(b,c) \notin R$

$\therefore$ R is not transitive

③ $(1,2) \in R \Rightarrow (2,1) \in R$

$(a,b) \in R \Rightarrow (b,a) \in R$

$\therefore$ R is symmetric

Q7.) Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x,y)$: x and y have same ~~number of~~ number of pages$\}$ is an equivalence relation.

Sol. ① $(x,y) \in R \Rightarrow$ x and y have same number of pages

$\Rightarrow$ y and x have same number of pages

$\Rightarrow (y,x) \in R$

$\therefore$ R is symmetric

② $(x,x) \in R \Rightarrow$ Same books have same number of pages

$\therefore$ R is reflexive

③ Let $(x,y) \in R$ and $(y,z) \in R$

x and y have same number of pages

y and z have same number of pages

then x and z also have same number of pages
$(x, z) \in R$
$\therefore$ R is transitive

R is reflexive, symmetric and transitive
$\therefore$ R is an equivalence relation.

Q8) Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a,b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

Sol. $A = \{1, 2, 3, 4, 5\}$
$R = \{(a, b) : |a-b| \text{ is even}\}$
$R = \{(1,1), (1,3), (3,1), (1,5), (5,1),$
$(2,2), (2,4), (4,2),$
$(3,5), (5,3)\}$

① $(1,1) \in R \quad \forall \; 1 \in R$
$(a,a) \in R \quad \forall \; a \in R$
$\therefore$ R is reflexive

② $(1,3) \in R$ and $(3,5) \in R \Rightarrow (1,5) \in R$
$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$
$\therefore$ R is ~~symmetric~~ transitive

③ $(1,5) \in R \Rightarrow (5,1) \in R$
$(a,b) \in R \Rightarrow (b,a) \in R$
$\therefore$ R is symmetric

Hence, R is an equivalence relation

① Let $(a,a) \in R \Rightarrow |a-a| = 0$ is even
$(a,a) \in R \;\; \forall \; a \in A$
$\therefore$ R is reflexive

② Let $(a,b) \in R \Rightarrow |a-b|$ is even
$\because |a-b| = |b-a|$
$\Rightarrow |b-a|$ is even
$\Rightarrow (b,a) \in R$
$\therefore$ R is symmetric

③ Let $(a,b) \in R$ and $(b,c) \in R \Rightarrow$
$a-b = \pm 2m, \; b-c = \pm 2n \; [m, n \in Z]$
$\Rightarrow a-b+b-c = \pm 2(m+n)$
$\Rightarrow a-c = \pm 2(m+n)$
$\Rightarrow |a-c|$ is even
$\Rightarrow (a,c) \in R$
$\therefore$ R is transitive
Hence, R is an equivalence relation

Elements of $\{1,3,5\}$ are related to each other as the difference between them is even

$|1-3| = 2$ is an even number
$|3-1| = 2$ is an even number

Elements of $\{2,4\}$ are related to each other as the difference between them is even

$|2-4| = 2$ is an even number
$|4-2| = 2$ is an even number

Elements of $\{1,3,5\}$ ~~and~~ ~~$\{2,4\}$~~ are not related to $\{2,4\}$ ~~each other~~ as the difference between them is not even

$|1-2| = 1$ is not an even number
$|5-2| = 3$ is not an even number

Q9.) Show that ~~each of~~ the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by $R = \{(a,b) : a = b\}$ is an equivalence relation.

Sol. ① Let $(a,a) \in R \Rightarrow a = a$

Every number is equal to itself

$\therefore$ R is reflexive

② Let $(a,b) \in R \Rightarrow a = b$
$\Rightarrow b = a$
$\Rightarrow (b,a) \in R$

$\therefore$ R is symmetric

③ Let $(a,b) \in R$ and $(b,c) \in R \Rightarrow a = b$ and $b = c$
$\Rightarrow a = c$
$\Rightarrow (a,c) \in R$

$\therefore$ R is transitive

$\therefore$ R is an equivalence relation

Find the set of all elements related to 1.

$(1, 1) \Rightarrow 1 = 1$

Only one element is related to 1

$[1] = \{1\}$ Ans

Q10.) Give an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(i) $A = \{1, 2, 3\}$

$R = \{(a, b) : a + b = 4\}, a, b \in A\}$

$R = \{(1, 3), (3, 1)\}$

R is symmetric but neither reflexive nor transitive

(ii) Transitive but neither reflexive nor symmetric.

(ii) $A = \{1, 2, 3\}$

$R = \{(a, b) : a > b\}, a, b \in A\}$

$R = \{(2, 1), (3, 2), (3, 1)\}$

R is transitive but neither reflexive nor symmetric

(iii) Reflexive and symmetric but not transitive

(iii) ~~$A = \{1, 2, 3\}$~~

~~$R = \{(a, b) : a + b \text{ is even}\}$~~

~~$R = \{(1, 1), (2, 2), (3, 3)\}$~~

~~R is~~

$R = \{(a, b) : a \text{ and } b \text{ have the same sign}\}, a, b \in Z\}$

R is reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(iv) $A = \{1, 2, 3\}$

$R = \{(a, b) : a \geq b\}, a, b \in A\}$

$R = \{(1,1), (2,2), (3,3), (2,1), (3,2), (3,1)\}$

R is reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

(v) $A = \{1, 2, 3\}$

$R = \{(x,y) : x \neq y\}$

$R = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$

R is symmetric and transitive but not reflexive.

Q11.) Show that the relation R in the set A of points in a plane given by $R = \{(P, Q)$: distance of the point P from the origin is same as the distance of the point Q from the origin$\}$, is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0,0)$ is the circle passing through P with origin as centre.

Sol. ① ~~Every~~ A point has same distance as itself from the origin

$(P, P) \in R \quad \forall P \in A$

$\therefore$ R is reflexive

② Let $(P, Q) \in R \Rightarrow$ P and Q have same distance from the origin

$\Rightarrow$ Q and P have same distance from the origin

$\Rightarrow (Q, P) \in R$

$\therefore$ R is symmetric

③ Let $(P, Q) \in R$ and $(Q, R) \in R$

P and Q have same distance from the origin

Q and R have same distance from the origin

then, P and R also have same distance from the origin

$\Rightarrow (P, R) \in R$

$\therefore$ R is transitive

$\therefore$ R is an equivalence relation

The set of all points related to a point $P \neq (0,0)$ is the circle passing through P with origin as centre as all points will be at equal distances from the origin.

Q13.) Show that the relation R defined in the set A of all polygons as $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$, is an equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Sol. ① A polygon is similar to itself

$(T_1, T_1) \in R \quad \forall\ T_1 \in A$

$\therefore$ R is reflexive

② Let $(T_1, T_2) \in R \Rightarrow T_1$ is similar to T_2

$\Rightarrow T_2$ is similar to T_1

$\Rightarrow (T_2, T_1) \in R$

$\therefore$ R is symmetric

③ Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$

T_1 is similar to T_2

T_2 is similar to T_3

then T_1 is also similar to T_3

$\Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive

$\therefore$ R is an equivalence relation

T_1 3, 4, 5

T_2 5, 12, 13

T_3 6, 8, 10

$$\frac{T_1}{T_3} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$$

T_1 and T_3 are similar
$\therefore T_1$ and T_2 are related

Q14.) Let L be the set of all lines in xy plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that ~~the~~ R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Sol. ① A line is parallel to itself
$(L_1, L_1) \in R \quad \forall \; L_1 \in L$
$\therefore R$ is reflexive

② Let $(L_1, L_2) \in R \Rightarrow L_1$ is parallel to L_2
$\Rightarrow L_2$ is parallel to L_1
$\Rightarrow (L_2, L_1) \in R$
$\therefore R$ is symmetric

③ Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$
L_1 is parallel to L_2
L_2 is parallel to L_3
then, L_1 is also parallel to L_3
$\Rightarrow (L_1, L_3) \in R$
$\therefore R$ is transitive

$\therefore R$ is an equivalence relation

$y = 2x + 4$
$y = mx + c$
$m = 2$
Two lines are parallel if they have same slope
Set of all lines related to the line $y = 2x + 4 \Rightarrow \{(x, y) : y = 2x + c, c \in R\}$

Q15.) Let R be the relation in the set $\{1,2,3,4\}$ given by $R = \{(1,2), (2,2), (1,1), (4,4), (1,3), (3,3), (3,2)\}$. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Sol. ① $(1,1), (2,2), (3,3)\ (4,4) \in R$

$(a,a) \in R \quad \forall a \in A$

$\therefore$ R is reflexive

② $(1,2) \in R$ but $(2,1) \notin R$

$(a,b) \in R$ but $(b,a) \notin R$

$\therefore$ R is not symmetric

③ ~~Let~~ $(1,3) \in R$ and $(3,2) \in R \Rightarrow (1,2) \in R$

$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

$\therefore$ R is transitive

hence, (B) R is reflexive and transitive but not symmetric Ans

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Q16.) Let R be the relation in the set N given by $R = \{(a,b) : a = b-2,\ b > 6\}$. Choose the correct answer.

(A) $(2,4) \in R$ (B) $(3,8) \in R$ (C) $(6,8) \in R$ (D) $(8,7) \in R$

Sol. $(6,8) \in R \Rightarrow 6 = 8 - 2$

$(a,b) \in R \Rightarrow a = b - 2 \ , \ b > 6$

$\therefore$ (C) $(6,8) \in R$ Ans

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★ EXERCISE - 1.2 (Continue)

Q1) Show that the function $f: R_* \rightarrow R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_*?

Sol. One-one

$x_1, x_2 \in R_*$ (D)

such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_2 = x_1$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

Onto Let $y \in R_*$ (C)

$f(x) = y \Rightarrow y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$

$f\left(\frac{1}{y}\right) \Rightarrow \frac{1}{\frac{1}{y}} \Rightarrow y$

For every $y \in R_*$ (C) $\exists$ an element $x = \frac{1}{y} \in R_*$ (D) such that $f\left(\frac{1}{y}\right) = y$

$\therefore$ f is onto.

$f: N \longrightarrow R_*$

One-one

$x_1, x_2 \in R_*$

such that $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_2 = x_1$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

~~Onto~~

~~Let $y \in N$~~

~~$f(x) = \frac{1}{x'} \Rightarrow y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \in R_*$~~

~~Let $y = -1$~~

~~$x = \frac{1}{-1} = -1 \notin$~~

<u>Onto</u>

Let $y \in R_*$

$f(x) = y \Rightarrow y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \in N$

Let $y = -1$

$x = \frac{1}{-1} = -1 \notin N$

Every element of co-domain R_* does not have a pre-image in domain N

$\therefore$ f is not onto

Hence, if $f: N \longrightarrow R_*$, f is one-one but not onto.

Q2.) Check the injectivity and surjectivity of the following functions:

(i) $f: N \longrightarrow N$ given by $f(x) = x^2$

(i) <u>One-one</u>

Let $x_1, x_2 \in N$

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is ~~one-one~~ injective

Onto

Let $y \in N_c$

$f(x) = y$

$\Rightarrow x^2 = y$

$\Rightarrow x = \sqrt{y}$

Let $y = 2$, $x = \sqrt{2} \notin N_D$

Thus, for every $y \in N_c$, $x = \sqrt{y} \notin N_D$

$\therefore$ f is not surjective

(iii) $f: R \longrightarrow R$ given by $f(x) = x^2$

(iii) One-one

Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

• $x_1 = x_2$, $x_1 = -x_2$

$\therefore$ f is not injective

Onto

Let $y \in R_c$

$f(x) = y$

$\Rightarrow x^2 = y$

$\Rightarrow x = \sqrt{y} \in R_D$

Let $y = -1$

$x = \sqrt{-1} \notin R_D$

Thus, for every $y \in R_c$, $x = \sqrt{y} \notin R_D$
$\therefore$ f is not surjective

(iv) $f: N \longrightarrow N$ given by $f(x) = x^3$

(iv) One-one

Let $x_1, x_2 \in N_D$
such that $f(x_1) = f(x_2)$
$\Rightarrow x_1^3 = x_2^3$
$\Rightarrow x_1 = x_2$
$\therefore$ f is ~~surjective~~ injective

Onto

Let $y \in N_C$
$f(x) = y$
$\Rightarrow x^3 = y$
$\Rightarrow x = \sqrt[3]{y} \in N_D$

Let $y = 2$, $x = \sqrt[3]{2} \notin N_D$
Thus, for every $y \in N_C$, $x = \sqrt[3]{y} \notin N_D$
$\therefore$ f is not surjective

(v) $f: Z \longrightarrow Z$ given by $f(x) = x^3$

(v) One-one

Let $x_1, x_2 \in Z_D$
such that $f(x_1) = f(x_2)$
$\Rightarrow x_1^3 = x_2^3$
$\Rightarrow x_1 = x_2$
$\therefore$ f is injective

Onto

Let $y \in Z_C$
$f(x) = y$
$\Rightarrow x^3 = y$

$\Rightarrow x = \sqrt[3]{y} \in Z_D$

Let $y = 2$, $x = \sqrt[3]{2} \notin Z_D$

Thus, for every $y \in Z_C$, $x = \sqrt[3]{y} \notin Z_D$

$\therefore$ f is not surjective

Q4) Show that the Modulus function $f: R \longrightarrow R$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Sol. One-one

$1, -1 \in R_D$

$|1| = 1$

$|-1| = 1$

Two elements have same image

$\therefore$ f is not one-one

Onto

Let $y \in R_C$

$f(x) = y$

$\Rightarrow |x| = y$

Let $y = -1 \in R_C$

$|x| = -1$

This is not possible since modulus function always gives positive value

$\therefore$ f is not onto

Q5.) Show that the Signum function $f: R \longrightarrow R$, given by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

is neither one-one nor onto.

Sol. <u>One-one</u>

$1, 2 \in R_D$

$f(1) = 1$

$F(2) = 1$

Two elements have same image

$\therefore$ f is not one-one

<u>Onto</u>

Range of signum function = $\{1, 0, -1\}$

$\{1, 0, -1\} \subset R_c$

Range $\subset$ Codomain

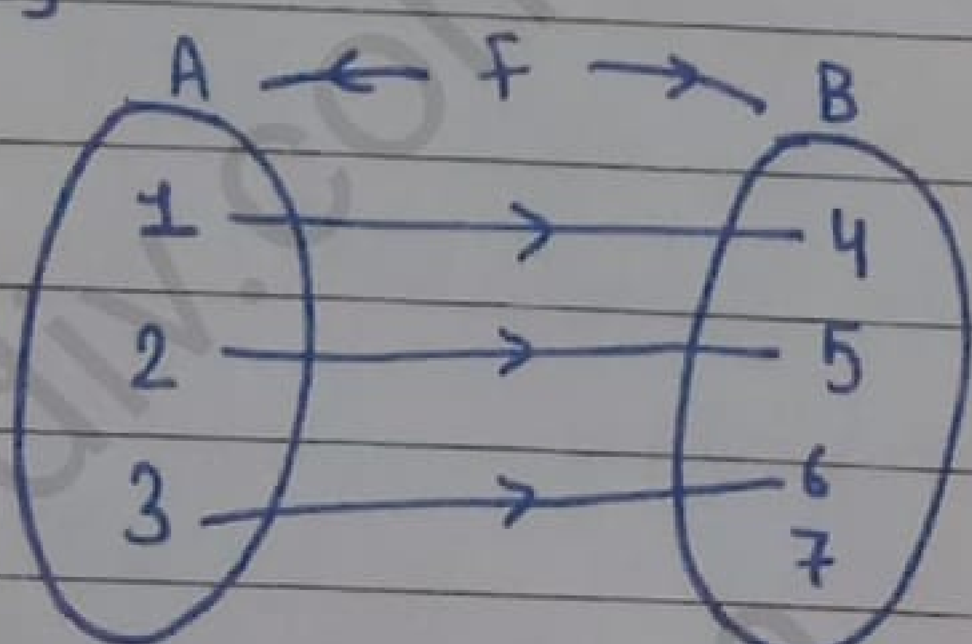
$\therefore$ f is not onto

Q6) Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1,4), (2,5), (3,6)\}$ be a function from A to B. Show that f is one-one.

Sol. $A = \{1, 2, 3\} \Rightarrow$ Domain

$B = \{4, 5, 6, 7\}$

Range $\Rightarrow \{4, 5, 6\}$



For every element of A, there is a unique distinct image of the element in B

$\therefore$ f is one-one

Q7) In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer

i) $f: R \rightarrow R$ defined by $f(x) = 3 - 4x$

(i) <u>One-one</u>

Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow \cancel{3} - \cancel{4}x_1 = \cancel{3} - \cancel{4}x_2$

$\Rightarrow x_1 = x_2$

Every element of R_D has distinct image in R_C

$\therefore$ f is one-one

<u>Onto</u>

Let $y \in R_C$

$f(x) = y$

$\Rightarrow 3 - 4x = y$

$\Rightarrow 4x = 3 - y$

$\Rightarrow x = \frac{3-y}{4} \in R_D$

For every $y \in R$ $\exists$ an element $x = \frac{3-y}{4} \in R_D$

such that $f\left(\frac{3-y}{4}\right) = 3 - \cancel{4} \times \left(\frac{3-y}{\cancel{4}}\right)$

$= \cancel{3} - \cancel{3} + y$

$= y$

$\therefore$ f is onto

Hence, f is bijective

(ii) $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$.

(ii) <u>One-one</u>

Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow \cancel{1} + x_1^2 = \cancel{1} + x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

$x_1 = x_2 \quad , \quad x_1 = -x_2$

$\therefore$ f is not one-one

Onto

$f(-1) = 1 + (-1)^2 = 2$

$f(0) = 1 + 0^2 = 1$

$f(2) = 1 + 2^2 = 5$

$f(3) = 1 + 3^2 = 10$

Co-domain = R

Range = $\{1, 2, 5, 10, \dots\}$

Range $\subset$ Co-domain

$\therefore$ f is not onto

Hence, f is not bijective

Q8.) Let A and B be sets, Show that $f: A \times B \longrightarrow B \times A$ such that $f(a,b) = (b,a)$ is bijective function.

Sol. One-one

Let (a_1, b_1) and $(a_2, b_2) \in (A \times B)$

such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\therefore$ f is one-one

Onto

Co-domain = $\{(b,a), (b_1, a_1), (b_2, a_2), \dots\}$

Range = $\{(b,a), (b_1, a_1), (b_2, a_2), \dots\}$

Range = Codomain

$\therefore$ f is onto

Hence, f is bijective function

Q10.) Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \longrightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Sol. <u>One-one</u>

Let $x_1, x_2 \in A$

such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow \cancel{x_1x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1x_2} - 2x_1 - 3x_2 + \cancel{6}$$

$$\Rightarrow -3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$\Rightarrow x_1 = x_2$$

$\therefore$ f is one-one

<u>Onto</u>

Let $y \in B$

$f(x) = y$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow yx - 3y = x - 2$$

$$\Rightarrow yx - x = 3y - 2$$

$$\Rightarrow x(y-1) = 3y - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1}$$

$$f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{\frac{3y - \cancel{2} - 2y + \cancel{2}}{\cancel{y-1}}}{\frac{\cancel{3y} - 2 - \cancel{3y} + 3}{\cancel{y-1}}} = y$$

For every $y \in B$ $\exists$ an element $x = \frac{3y-2}{y-1} \in A$ such that $f(x) = y$

$\therefore$ f is onto

Q11.) Let $f: R \to R$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

Sol. One-one

Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^4 = x_2^4$

$\Rightarrow x_1 = \sqrt[4]{x_2^4}$

$\Rightarrow x_1 = \pm x_2$

$\therefore$ f is not one-one

Onto

Let $y \in R_C$

$f(x) = y$

$\Rightarrow x^4 = y$

$\Rightarrow x = \sqrt[4]{y} \in R_D$

Let $y = -1$, $x = \sqrt[4]{-1} \notin R_D$

$\therefore$ f is not onto

$\therefore$ (D) f is neither one-one nor onto Ans

Q12.) Let $f: R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto.

Sol. One-one

Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

Onto

Let $y \in R_C$
$f(x) = y$
$\Rightarrow 3x = y$
$\Rightarrow x = \frac{y}{3} \in R_D$

$f\left(\frac{y}{3}\right) = 3 \times \frac{y}{3} = y$

For every $y \in R_C$, $\exists$ an element $x = \frac{y}{3}$ such that $f(x) = y$

$\therefore$ f is onto

$\therefore$ (A) f is One-one onto Ans

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☆ MISCELLANEOUS EXERCISE ON CHAPTER 1

Q1) Show that the function $f: R \longrightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one one and onto function.

Sol. $f(x) = \begin{cases} \frac{x}{1+x}, & x \geq 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$

Let $\{x \in R : -1 < x < 1\} = B$

<u>One-one</u>

(i) Let $x_1, x_2 \in R$, $x_1, x_2 \geq 0$
such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{1+x_1} = \frac{x_2}{1+x_2}$$

$$\Rightarrow x_1 + \cancel{x_1 x_2} = x_2 + \cancel{x_1 x_2}$$

$$\Rightarrow x_1 = x_2$$

(ii) Let $x_1, x_2 \in R$, $x_1, x_2 < 0$
such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2}$$

$$\Rightarrow x_1 - \cancel{x_1 x_2} = x_2 - \cancel{x_1 x_2}$$

$$\Rightarrow x_1 = x_2$$

$\therefore$ f is one-one

<u>Onto</u>

(i) Let $y \in B$,
$f(x) = y$, $x \geq 0$

$$\Rightarrow \frac{x}{1+x} = y$$

(ii) Let $y \in B$
$f(x) = y$, $x < 0$

$$\Rightarrow \frac{x}{1-x} = y$$

$\Rightarrow x = y + yx$

$\Rightarrow x - yx = y$

$\Rightarrow x(1-y) = y$

$\Rightarrow x = \frac{y}{1-y}$

$x \geq 0$

$\Rightarrow \frac{y}{1-y} \geq 0$

$\Rightarrow \frac{y}{y-1} \leq 0$

$y \in [0, 1)$

$\Rightarrow x = y - yx$

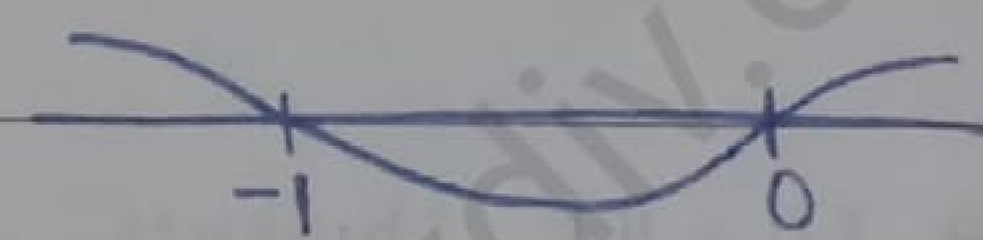
$\Rightarrow y = x + yx$

$\Rightarrow x = \frac{y}{1+y}$

$x < 0$

$\Rightarrow \frac{y}{1+y} < 0$

$\Rightarrow \frac{y}{y+1} < 0$



$y \in (-1, 0)$

$y \in (-1, 1)$

Range = Codomain

$\therefore$ f is onto

Q2.) Show that the function $f: R \rightarrow R$ given by $f(x) = x^3$ is injective.

Sol. Let $x_1, x_2 \in R_D$

such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

f is injective

Hence, ~~proved~~ proved

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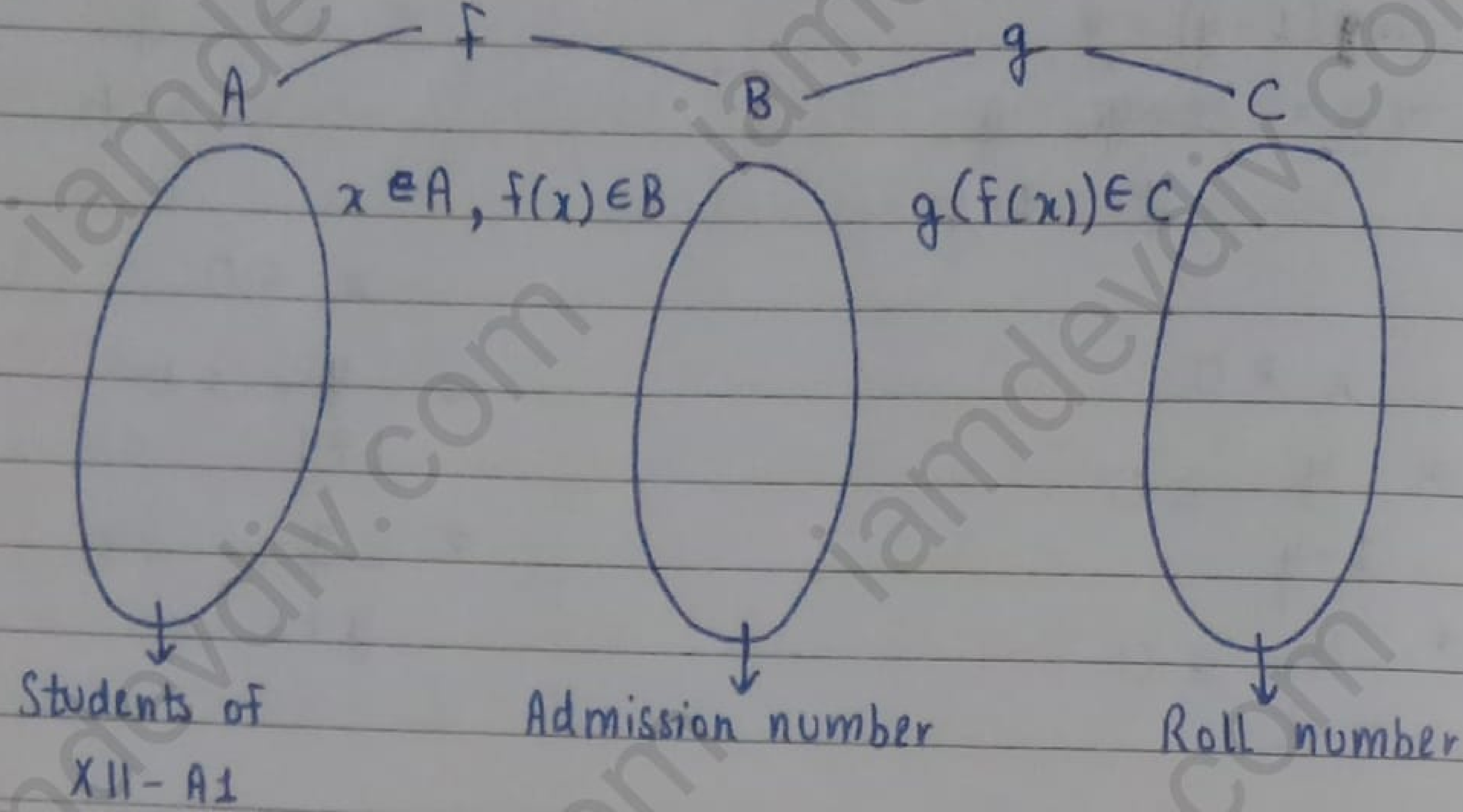
★ COMPOSITION OF TWO FUNCTIONS



$A \Rightarrow$ XII - A1 students

$B \Rightarrow$ Admission number of XII students

$C \Rightarrow$ Roll number of XII students

$f: A \longrightarrow B$ and $g: B \rightarrow C$

Range of $f \subset B$

$g[f(x)] = gof(x)$

$gof: A \longrightarrow C$

$(gof)(x) = g[f(x)]$

The composition of f and g is denoted by gof and defined by

Q $f: \{2, 3, 4, 5\} \longrightarrow \{3, 4, 5, 9\}$

$g: \{3, 4, 5, 9\} \longrightarrow \{7, 11, 15\}$

$f(2) = 3$	$g(3) = 7$
$f(3) = 4$	$g(4) = 11$
$f(4) = 5$	$g(9) = 15$
$f(5) = 9$	$g(5) = 11$

Find gof.

Sol. $gof(2) = g(f(2)) = g(3) = 7$

$gof(3) = g[f(3)] = g(4) = 11$

$gof(4) = g[f(4)] = g(5) = 11$

$gof(5) = g[f(5)] = g(9) = 15$

Q $f: R \to R$ $\quad$ $g: R \to R$

$f(x) = x^3$ $\quad$ $g(x) = 2x + 3$

Is $fog = gof$?

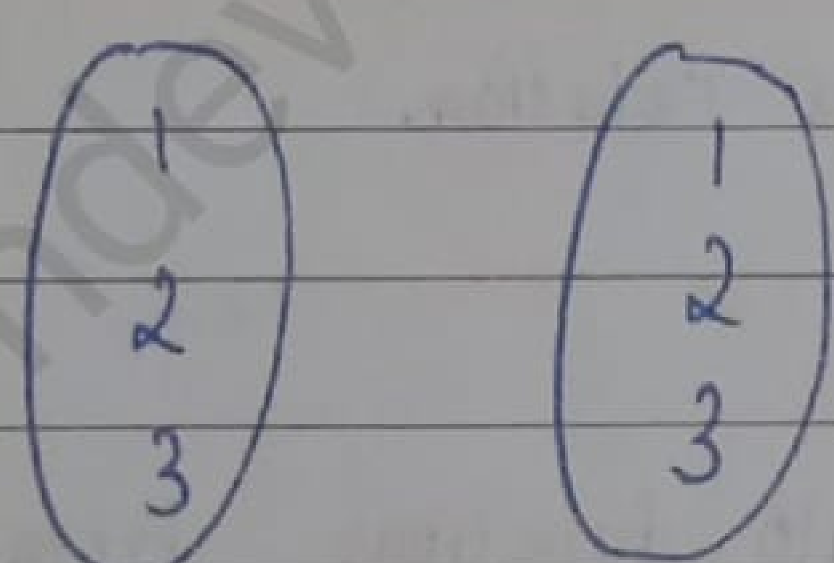
Sol. $fog(x) = f[g(x)]$
$= f(2x+3)$
$= (2x+3)^3$

$gof(x) = g[f(x)]$
$= g(x^3)$
$= 2x^3 + 3$

$\therefore$ No, $fog \neq gof$

Q Set A has n elements
How many onto functions are defined on A to itself?

Sol. If



3 elements

$\boxed{3}\boxed{2}\boxed{1} = 6 = 3!$

$\therefore$ for n elements $\Rightarrow$ $n!$ onto functions are defined
$\hookrightarrow$ same goes for one-one functions

Q How many functions can be defined from a set of m elements to another set of n elements?

Sol. n^m

15/5/23 ★ MISCELLANEOUS EXERCISE ON CHAPTER 1 (Continue)

Q3) Given a non-empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows:
For Subsets, A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on P(X)? Justify your answer.

Sol ① Every set is a subset of itself

$A \subset A$, $B \subset B$

$\therefore$ R is reflexive

② $A \subset B$

$B \not\subset A$

$\therefore$ R is not symmetric

③ $A \subset B$ and $B \subset C \Rightarrow A \subset C$

$\therefore$ R is transitive

Hence, R is not an equivalence relation.

Q4.) Find the number of all onto functions from the set $\{1, 2, 3, \ldots, n\}$ to itself.

Sol. Pre-images $= n \times (n-1) \times (n-2) \ldots$

$= n!$ Ans

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Q5.) Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g : A \longrightarrow B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1$, $x \in A$. Are f and g equal?

Sol. $f(-1) \Rightarrow (-1)^2 - (-1) \Rightarrow 1 + 1 \Rightarrow 2$

$g(-1) \Rightarrow 2\left|-1 - \frac{1}{2}\right| - 1 \Rightarrow 2\left|\frac{-3}{2}\right| - 1 \Rightarrow \cancel{2} \times \frac{3}{\cancel{2}} - 1 \Rightarrow 2$

$f(0) \Rightarrow 0^2 - 0 \Rightarrow 0$

$g(0) \Rightarrow 2\left|0 - \frac{1}{2}\right| - 1 \Rightarrow \cancel{2} \times \frac{1}{\cancel{2}} - 1 \Rightarrow 0$

$f(1) \Rightarrow 1^2 - 1 \Rightarrow 0$

$g(1) \Rightarrow 2\left|1 - \frac{1}{2}\right| - 1 \Rightarrow \cancel{2} \times \frac{1}{\cancel{2}} - 1 \Rightarrow 0$

$f(2) \Rightarrow 2^2 - 2 \Rightarrow 2$

$g(2) \Rightarrow 2\left|2 - \frac{1}{2}\right| - 1 \Rightarrow 2\left|\frac{3}{2}\right| - 1 \Rightarrow \cancel{2} \times \frac{3}{\cancel{2}} - 1 \Rightarrow 2$

$\because f(a) = g(a) \ \forall \ a \in A$

Hence, f and g are equal.

Q6.) Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

Sol. $R = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (3,1)\}$

$(a, a) \in A \ \forall \ a \in A \Rightarrow$ Reflexive

$(a, b) \in A \Rightarrow (b, a) \in A \Rightarrow$ Symmetric

$(3,1) \in R$ and $(1,2) \in R$ but $(3,2) \notin R \Rightarrow$ Not transitive

$\therefore$ (A) 1 Ans

Q7) Let $A = \{1,2,3\}$. Then number of equivalence relations containing $(1,2)$ is

(A) 1 (B) 2 (C) 3 (D) 4

Sol. $R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

$(a,a) \in R_1 \ \forall \ a \in A \Rightarrow$ Reflexive
$(a,b) \in R_1 \Rightarrow (b,a) \in R_1 \Rightarrow$ Symmetric
$(a,b) \in R_1$ and $(b,c) \in R_1 \Rightarrow (a,c) \in R_1 \Rightarrow$ Transitive

$R_2 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$(a,a) \in R_2 \ \forall \ a \in A \Rightarrow$ Reflexive
$(a,b) \in R_2 \Rightarrow (b,a) \in R_2 \Rightarrow$ Symmetric
$(a,b) \in R_2$ and $(b,c) \in R_2 \Rightarrow (a,c) \in R_2 \Rightarrow$ Transitive

R_1 and R_2 are equivalence relations
$\therefore$ (B) 2 Ans
